Trigonometric Function Graphing Sheet

1 Sine ($y = \sin x$)

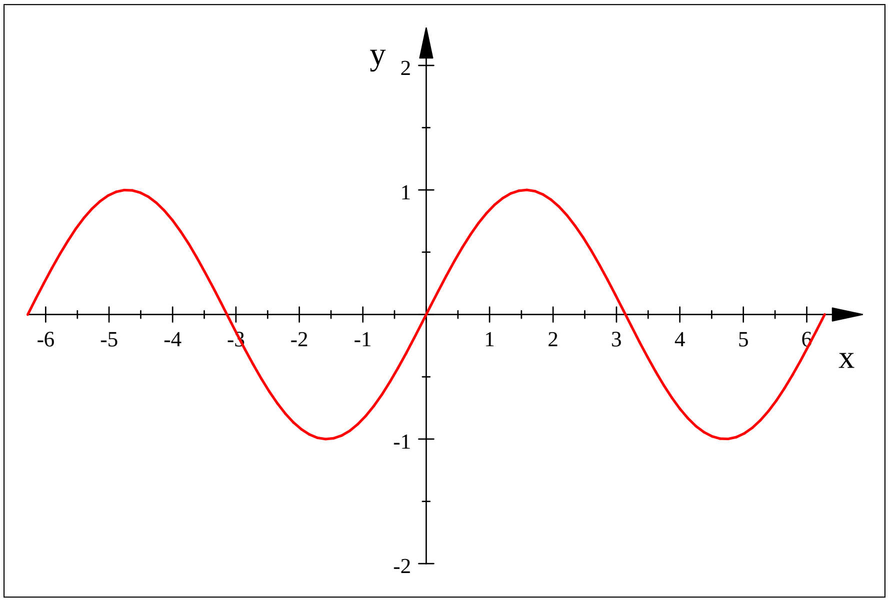
The value of sine at common angles:

$x =$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x =$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

The values of sine that we typically use to draw its graph:

$x =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x =$	0	1	0	-1	0

The graph of $y = \sin x$ looks like: (2 periods are shown)



A transformation of $y = \sin x$ is of the following form:

$$y = A \sin\left(B\left(x - C\right)\right) + D$$

where:

- $|A|$ is the amplitude of the graph,

- the period of the graph is $P = \frac{2\pi}{B}$,
- the phase shift of the graph is C,
- and the vertical translation is D.

Some facts:

- The domain is $(-\infty, \infty)$,
- the range is $(-A + D, A + D)$.

A geometric interpretation of the equation (these descriptions are transformations of $y = \sin x$):

- The amplitude is defined to be $\frac{1}{2}|y_{\max} - y_{\min}|$. If $|A| > 1$, the graph gets stretched taller in the $y - direction$; if $0 < |A| < 1$, then the graph gets shrunk shorter in the $y - direction$; lastly if $|A| = 1$, the graph does not get stretched or shrunk in the vertical direction. (We do not consider the case if $A = 0$ because, if $A = 0$, the graph is just the line $y = 0$.) Lastly, if $A < 0$, then the graph is reflected over the $x - axis$.

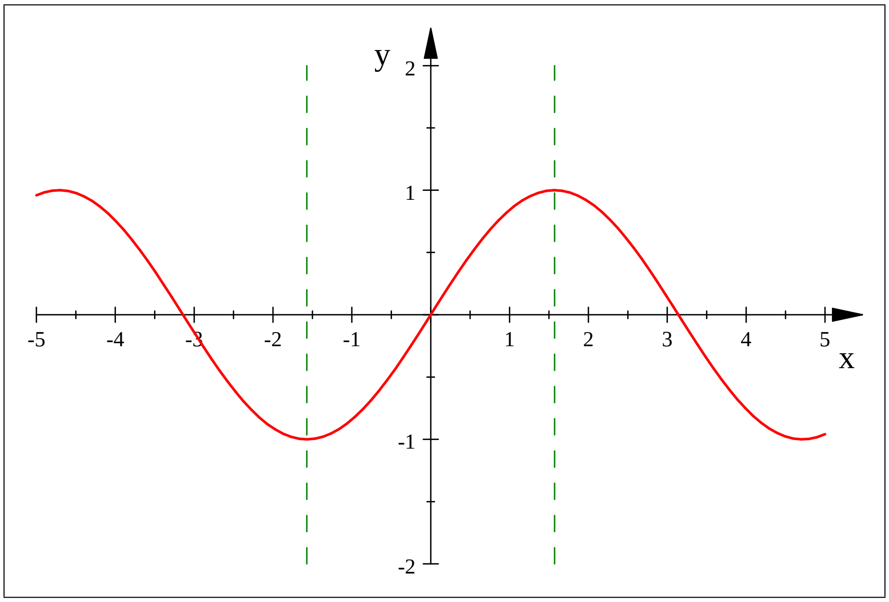
- The period is how long the graph takes to complete one cycle. We can assume B to be positive since sine is an odd function (i.e. $\sin(-x) = -\sin(x)$). The standard period of sine is 2π. If $B > 1$ the period is shrunk, and if $B < 1$ the period is expanded. Again here we take $B \neq 0$ as otherwise the graph would be a constant. If $B = 1$, the period is unchanged. When trying to graph $y = \sin Bx$ you can use the following table:

$x =$	0	$\frac{\frac{\pi}{2}}{B}$	$\frac{\pi}{B}$	$\frac{\frac{3\pi}{2}}{B}$	$\frac{2\pi}{B}$
$\sin Bx =$	0	1	0	-1	0

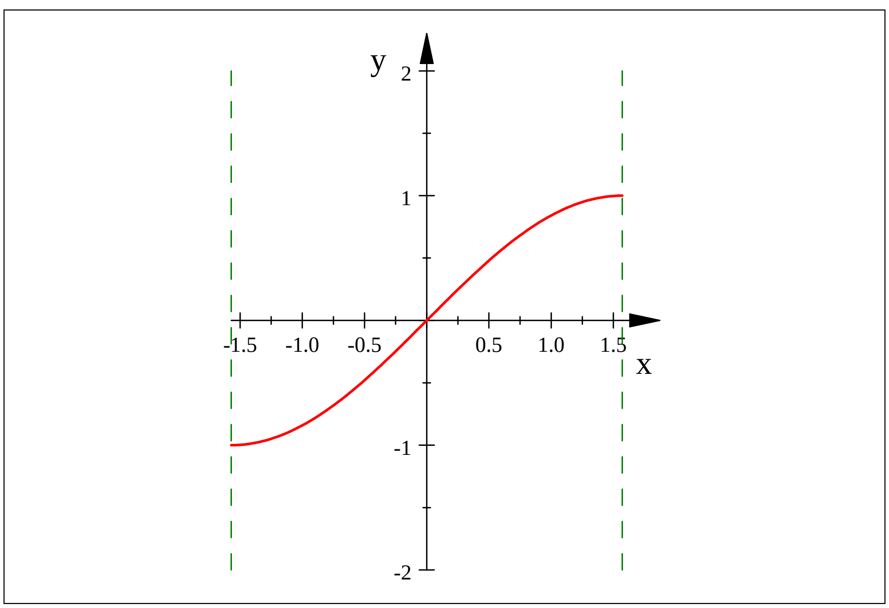
- The phase shift C is a horizontal translation C units to the right if $C > 0$, and a horizontal translation C units to the left if $C < 0$. If $C = 0$, the graph remains unchanged.

- The vertical translation D moves the graph D units upward if $D > 0$, D units downward if $D < 0$, and if $D = 0$, the graph remains unchanged.

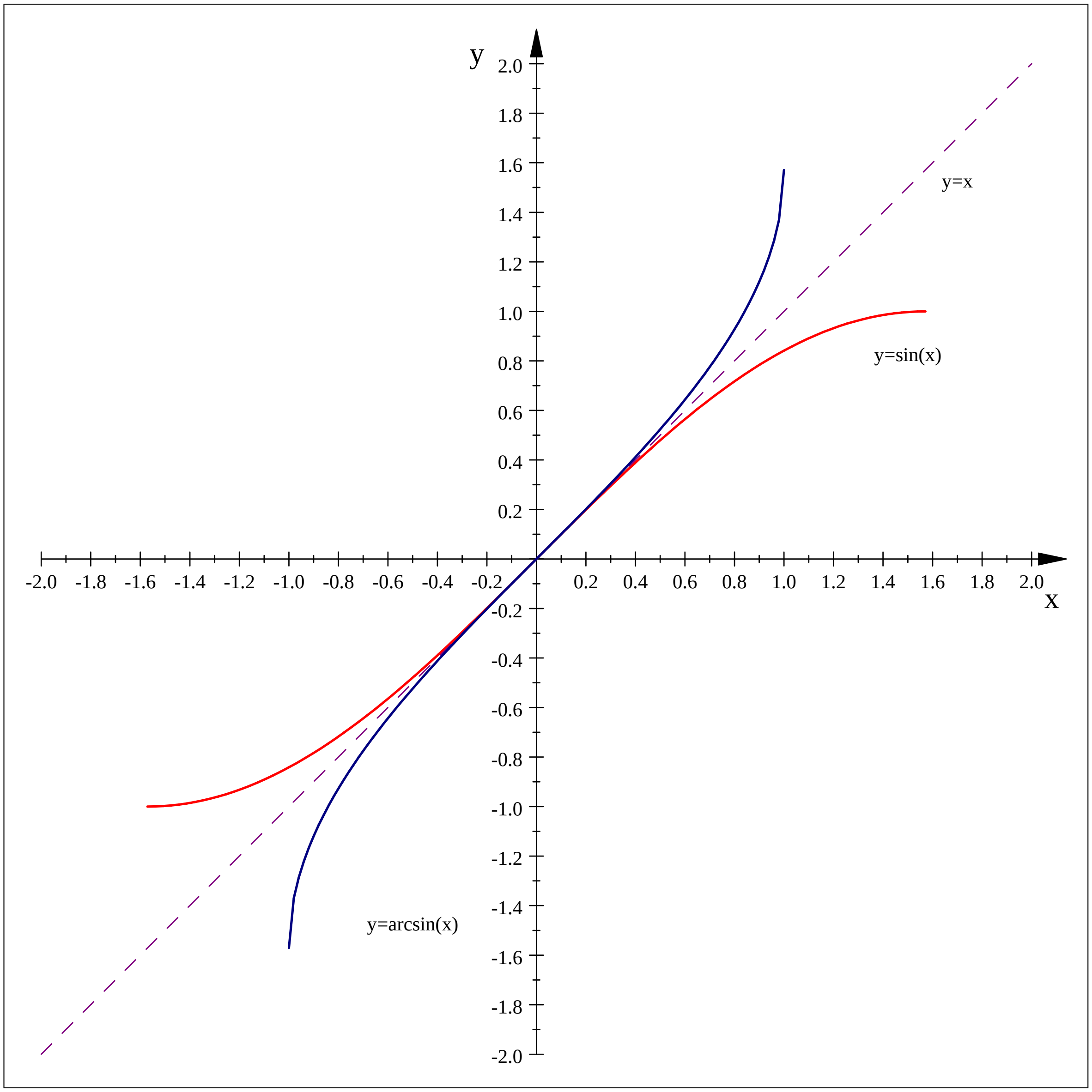
To invert sine we have to restrict to a domain where sine is *one − to − one*. Observe the following graph:



The green lines above cut the graph at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. Notice that inside these two green lines the graph is $one - to - one$ and increasing. So lets ignore everything outside of these green lines:



Now the graph is invertible, so to invert it let's draw the diagonal $y = x$ and reflect the above graph across the diagonal:



Notice that when we restrict sine to be one-to-one, the three points $\left(-\frac{\pi}{2}, -1\right), (0,0), \left(\frac{\pi}{2}, 1\right)$ are the two endpoints and the midpoint. Since an inverse function basically switches the x and y coordinates, these three points become $\left(-1, -\frac{\pi}{2}\right), (0,0), \left(1, \frac{\pi}{2}\right)$ for $y = \arcsin x$.

Some properties of $y = \arcsin x$:

- It is one-to-one.

- It is increasing.
- It is odd.
- Its domain is $[-1, 1]$.
- Its range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

2 Cosine $(y = \cos x)$

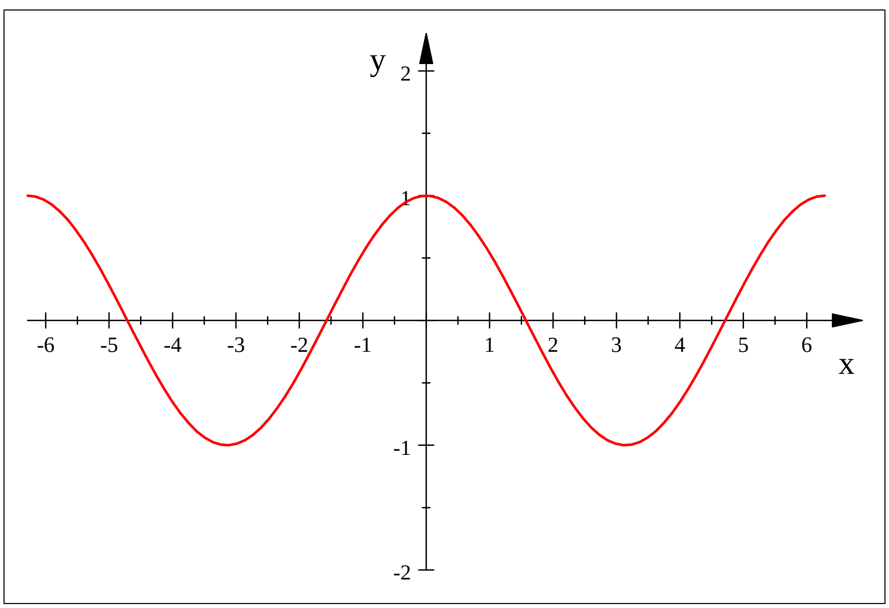
The value of cosine at common angles:

$x =$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos x =$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

The values of sine that we typically use to draw its graph:

$x =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x =$	1	0	-1	0	1

The graph of $y = \cos x$ looks like: (2 periods are shown)



A transformation of $y = \cos x$ is of the following form:

$$y = A\cos\left(B\left(x - C\right)\right) + D$$

where:

- $|A|$ is the amplitude of the graph,
- the period of the graph is $P = \frac{2\pi}{B}$,
- the phase shift of the graph is C,
- and the vertical translation is D.

Some facts:

- The domain is $(-\infty, \infty)$,
- the range is $(-A + D, A + D)$.

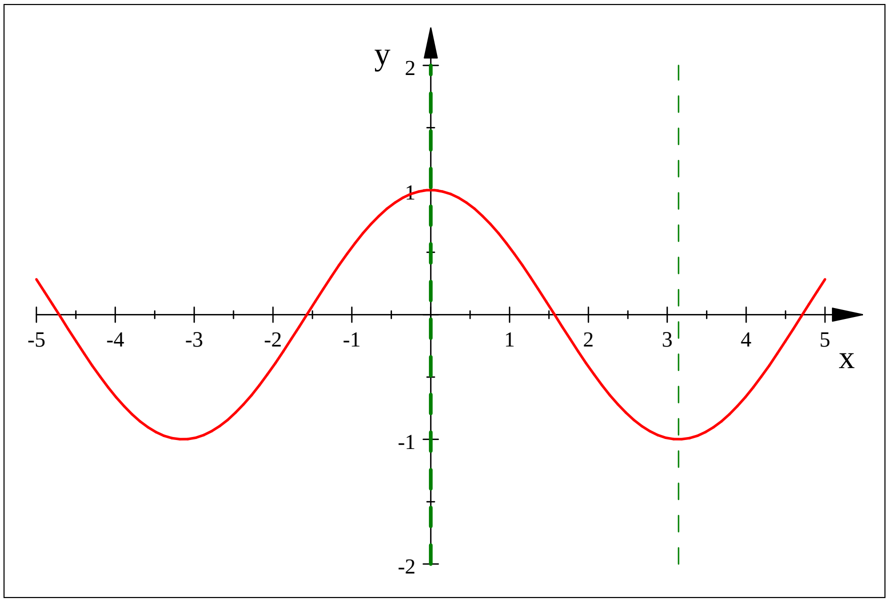
A geometric interpretation of the equation (these descriptions are transformations of $y = \cos x$):

- The amplitude is defined to be $\frac{1}{2}\left|y_{\max} - y_{\min}\right|$. If $|A| > 1$, the graph gets stretched taller in the $y - direction$; if $0 < |A| < 1$, then the graph gets shrunk shorter in the $y - direction$; lastly if $|A| = 1$, the graph does not get stretched or shrunk in the vertical direction. (We do not consider the case if $A = 0$ because, if $A = 0$, the graph is just the line $y = 0$.) Lastly, if $A < 0$, then the graph is reflected over the $x - axis$.
- The period is how long the graph takes to complete one cycle. The standard period of cosine is 2π. We can assume B to be positive since cosine is an even function (i.e. $\cos(-x) = \cos(x)$). If $B > 1$ the period is shrunk, and if $B < 1$ the period is expanded. Again here we take $B \neq 0$ as otherwise the graph would be a constant. If $B = 1$, the period is unchanged. When trying to graph $y = \cos Bx$ you can use the following table:

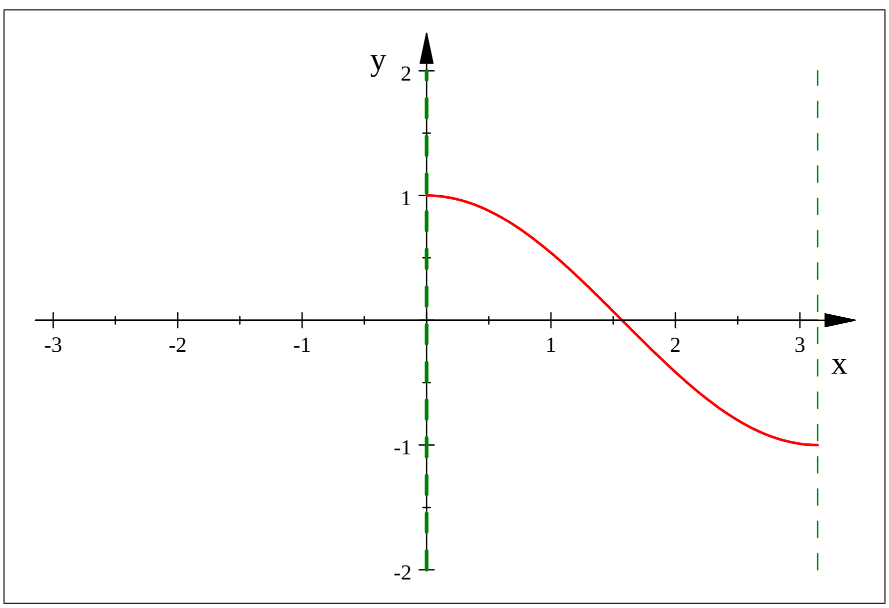
$x =$	0	$\frac{\frac{\pi}{2}}{B}$	$\frac{\pi}{B}$	$\frac{\frac{3\pi}{2}}{B}$	$\frac{2\pi}{B}$
$\cos Bx =$	1	0	-1	0	1

- The phase shift C is a horizontal translation C units to the right if $C > 0$, and a horizontal translation C units to the left if $C < 0$. If $C = 0$, the graph remains unchanged.
- The vertical translation D moves the graph D units upward if $D > 0$, D units downward if $D < 0$, and if $D = 0$, the graph remains unchanged.

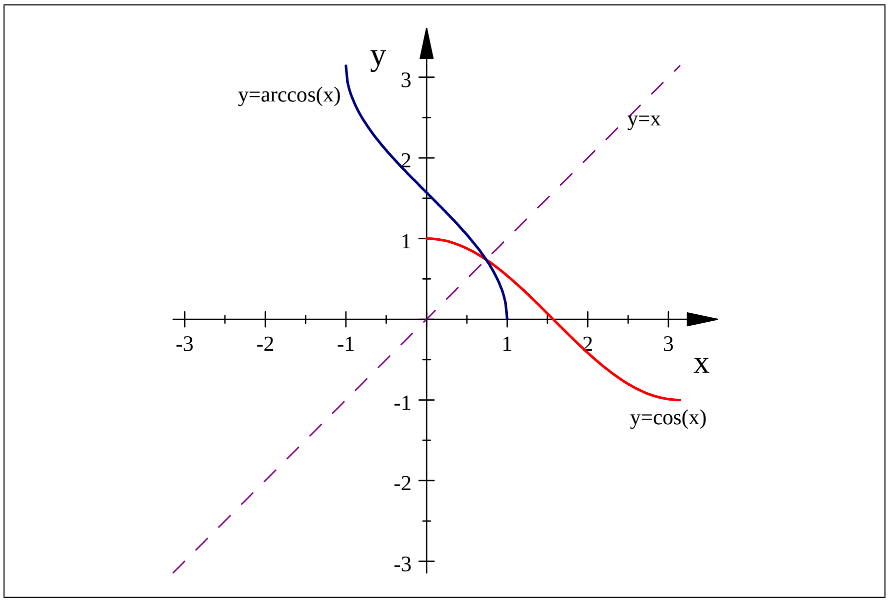
To invert cosine we have to restrict to a domain where cosine is $one{-}to{-}one$. Observe the following graph:



The green lines above cut the graph at $x = 0$ and $x = \pi$. Notice that inside these two green lines the graph is $one - to - one$ and decreasing. So lets ignore everything outside of these green lines:



Now the graph is invertible, so to invert it let's draw the diagonal $y = x$ and reflect the above graph across the diagonal:



Notice that when we restrict cosine to be one-to-one, the three points $(0,1)$, $\left(\frac{\pi}{2},0\right)$, $(\pi,-1)$ are the two endpoints and the midpoint. Since an inverse function basically switches the x and y coordinates, these three points become $(1,0)$, $\left(0,\frac{\pi}{2}\right)$, $(-1,\pi)$ for $y=\arccos x$.

Some properties of $y=\arccos x$:

- It is one-to-one.
- It is decreasing.
- Its domain is $[-1,1]$.
- Its range is $[0,\pi]$.

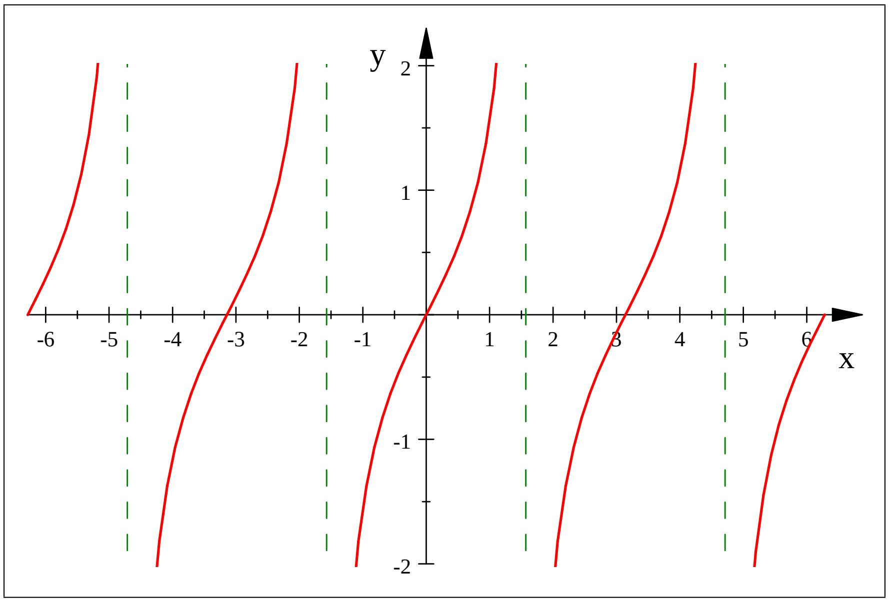
3 Tangent $(y=\tan x)$

The value of tangent at common angles:

$x=$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan x=$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$undefined$

To graph tangent we usually plot the point $(0,0)$ and use its asymptotes at $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$ to help keep the shape. Other points we can use are $\left(-\frac{\pi}{4},-1\right)$ and $\left(\frac{\pi}{4},1\right)$. Then using the fact that tangent is periodic we extend it to the rest of the real line.

The graph of $y=\tan x$ looks like:



Notice that $y = \tan x$ has vertical asymptotes at $x = \frac{n\pi}{2}$ where n is an odd integer $(..., -5, -3, -1, 1, 3, 5, ...)$.

A transformation of $y = \tan x$ is of the following form:

$$y = A \tan\left(B\left(x - C\right)\right) + D$$

where:

- A stretches (or shrinks) the graph,
- the period of the graph is $P = \frac{\pi}{B}$,
- the phase shift of the graph is C,
- and the vertical translation is D.

Some facts:

- The domain is $\mathcal{D} = \left\{x \in \mathbb{R} \mid x \neq \frac{n\pi}{2} \text{ where } n \text{ is an odd integer } (..., -3, -1, 1, 3, ...)\right\}$,
- the range is $(-\infty, \infty)$.

A geometric interpretation of the equation (these descriptions are transformations of $y = \tan x$):

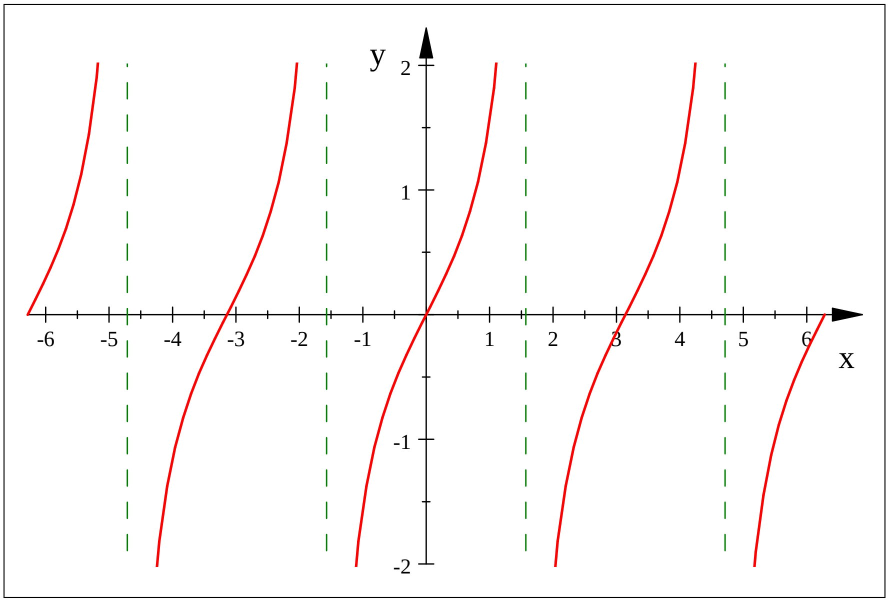
- If $|A| > 1$, the graph gets stretched taller in the $y-direction$; if $0 < |A| < 1$, then the graph gets shrunk shorter in the $y-direction$; lastly if $|A| = 1$, the graph does not get stretched or shrunk in the vertical direction. (We do not consider the case if $A = 0$ because, if $A = 0$, the graph is just the line $y = 0$.) Lastly, if $A < 0$, then the graph is reflected over the $x - axis$.

- The period is how long the graph takes to complete one cycle. The standard period of tangent is π. The standard position of the period is from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$. We can assume B to be positive since tangent is an odd function (i.e. $\tan(-x) = -\tan(x)$). If $B > 1$ the period is shrunk, and if $B < 1$ the period is expanded. Again here we take $B \neq 0$ as otherwise the graph would be a constant. If $B = 1$, the period is unchanged. When trying to graph $y = \tan Bx$ you can just alter the vertical asymptotes $y = \frac{n\pi}{2}$ to be $y = \frac{n\pi}{2B}$, then just graph the point on the $x - axis$ and use the asymptotes to keep the shape; also, you can use the points $\left(\frac{-\frac{\pi}{4}}{B}, -1\right)$ and $\left(\frac{\frac{\pi}{4}}{B}, 1\right)$.

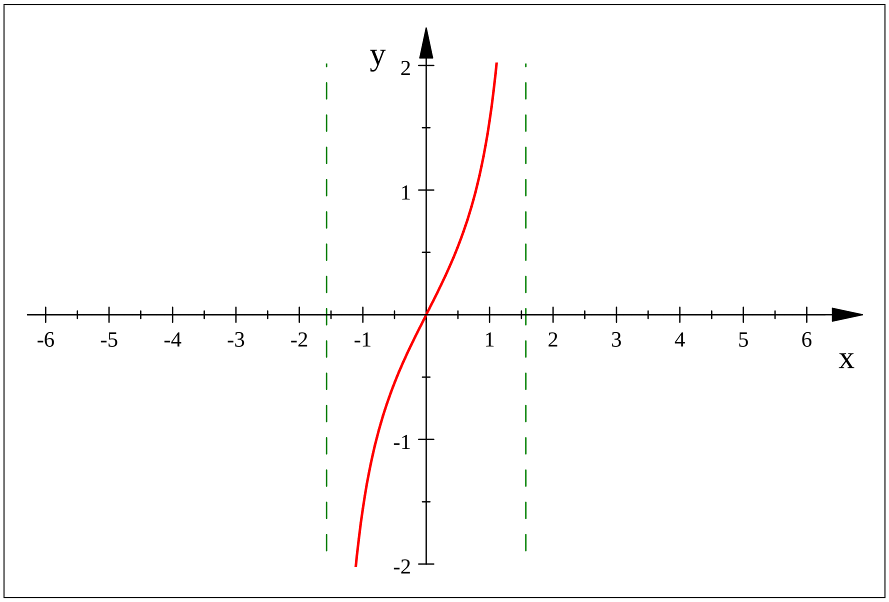
- The phase shift C is a horizontal translation C units to the right if $C > 0$, and a horizontal translation C units to the left if $C < 0$. If $C = 0$, the graph remains unchanged. Make sure that you also translate the asymptotes with the graph!

- The vertical translation D moves the graph D units upward if $D > 0$, D units downward if $D < 0$, and if $D = 0$, the graph remains unchanged.

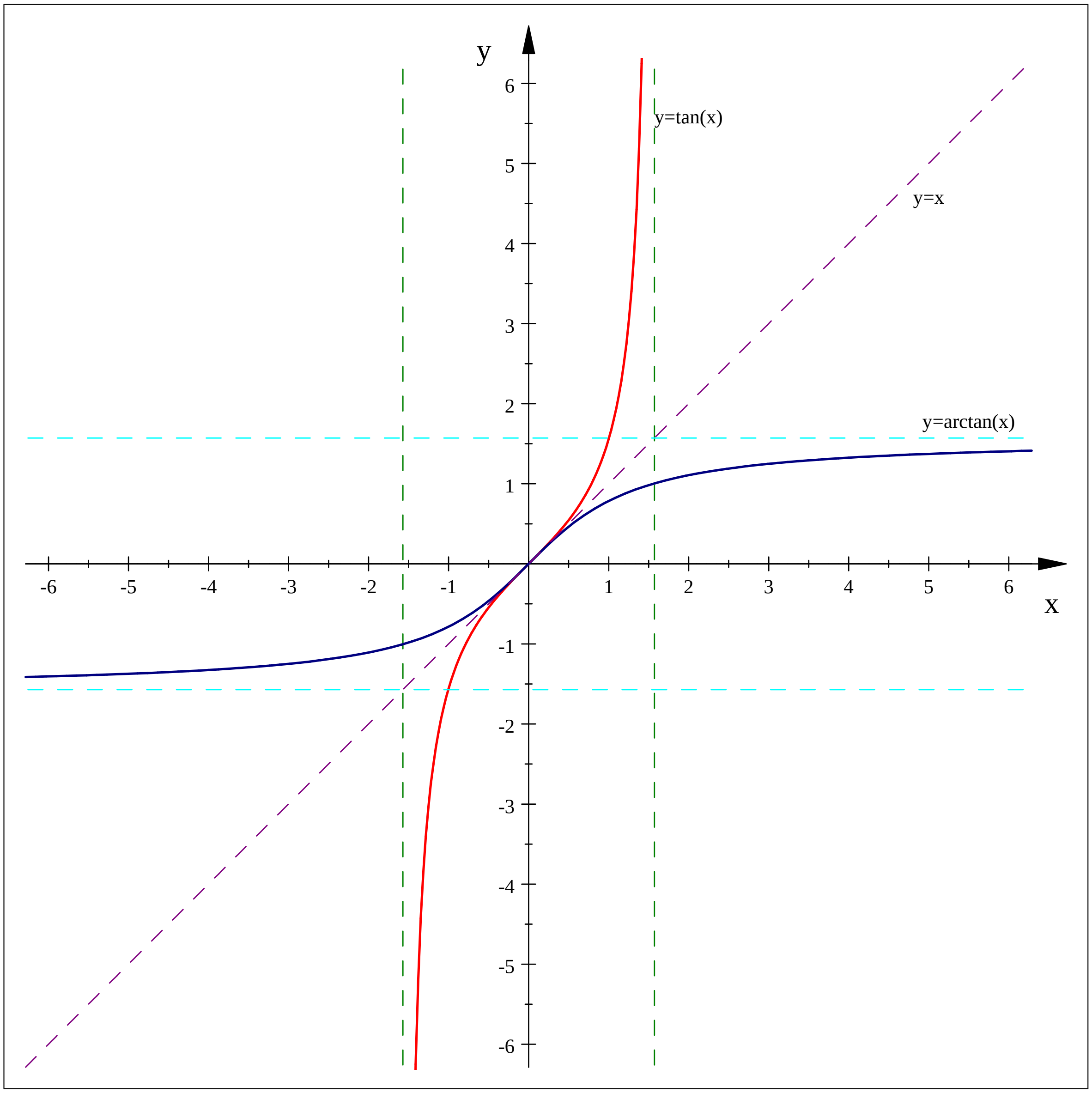
To invert tangent we have to restrict to a domain where tangent is $one - to - one$. Observe the following graph:



Notice that inside each pair of adjacent asymptotes the graph is *one−to−one* and increasing. Consider the middle two asymptotes. So lets ignore everything outside of these asymptotes:



Now the graph is invertible, so to invert it let's draw the diagonal $y = x$ and reflect the above graph (and its asymptotes!) across the diagonal:



Notice that when we restrict tangent to be one-to-one, the point of intersection with the $x-axis$ stays the same, so we can draw $y = \arctan x$ the same way as $y = \tan x$, using the asymptotes (which are the asymptotes of $y = \tan(x)$ reflected about the line $y = x$). Since an inverse function basically switches the x and y coordinates, the three points above become $\left(-1, -\frac{\pi}{4}\right), (0,0), \left(1, \frac{\pi}{4}\right)$ for $y = \arctan x$.

Some properties of $y = \arctan x$:

- It is one-to-one.

- It is increasing.
- It is odd.
- Its domain is $(-\infty, \infty)$.
- Its range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

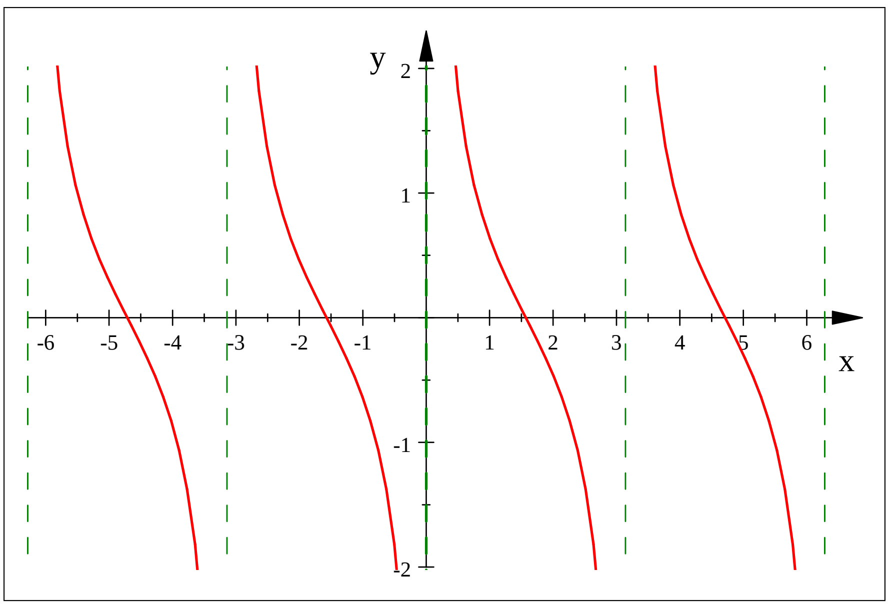
4 Cotangent $(y = \cot x)$

The value of cotangent at common angles:

$x =$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cot x =$	*undefined*	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

To graph cotangent we usually plot the point $\left(\frac{\pi}{2}, 0\right)$ and use its asymptotes at $x = 0$ and $x = \pi$ to help keep the shape. Other points we can use are $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{3\pi}{4}, -1\right)$. Then using the fact that cotangent is periodic we extend it to the rest of the real line.

The graph of $y = \cot x$ looks like:



Notice that $y = \cot x$ has vertical asymptotes at $x = n\pi$ where n is an integer $(..., -2, -1, 0, 1, 2, ...)$.

A transformation of $y = \cot x$ is of the following form:

$$y = A\cot\left(B\left(x - C\right)\right) + D$$

where:

- A stretches (or shrinks) the graph,
- the period of the graph is $P = \frac{\pi}{B}$,
- the phase shift of the graph is C,
- and the vertical translation is D.

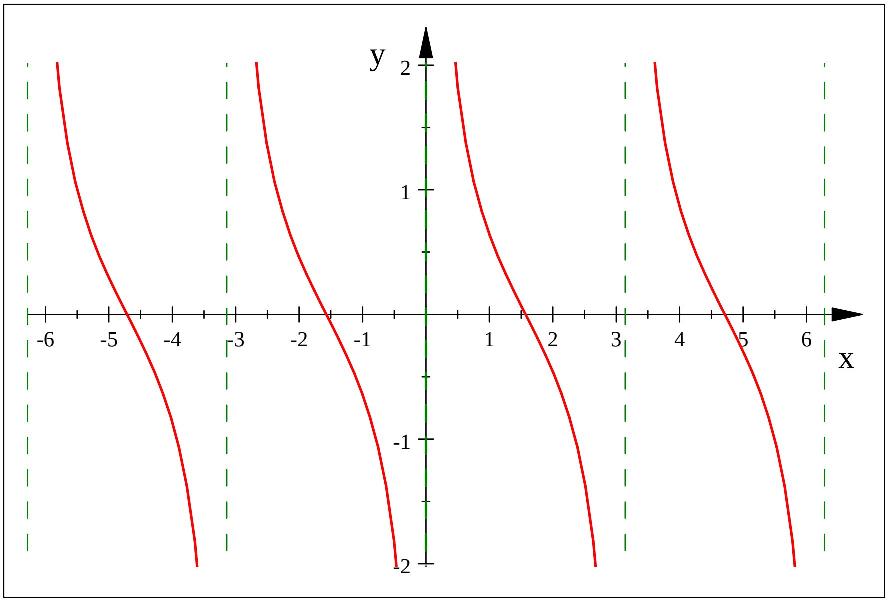
Some facts:

- The domain is $\mathcal{D} = \{x \in \mathbb{R} \mid x \neq n\pi \text{ where } n \text{ is an integer } (..., -2, -1, 0, 1, 2, ...)\}$,
- the range is $(-\infty, \infty)$.

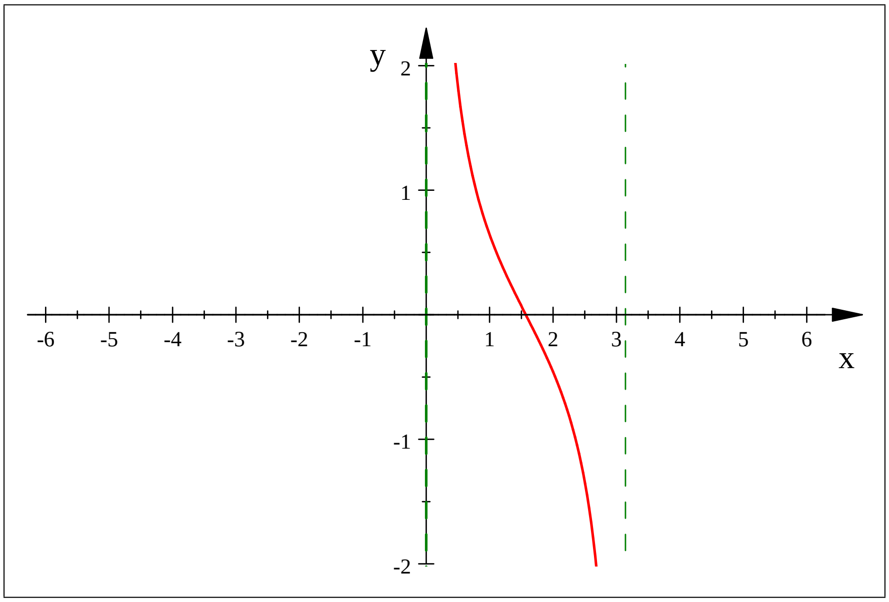
A geometric interpretation of the equation (these descriptions are transformations of $y = \cot x$):

- If $|A| > 1$, the graph gets stretched taller in the $y - direction$; if $0 < |A| < 1$, then the graph gets shrunk shorter in the $y - direction$; lastly if $|A| = 1$, the graph does not get stretched or shrunk in the vertical direction. (We do not consider the case if $A = 0$ because, if $A = 0$, the graph is just the line $y = 0$.) Lastly, if $A < 0$, then the graph is reflected over the $x - axis$.
- The period is how long the graph takes to complete one cycle. The standard period of cotangent is π. The standard position of the period is from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$. We can assume B to be positive since cotangent is an odd function (i.e. $\cot(-x) = -\cot(x)$). If $B > 1$ the period is shrunk, and if $B < 1$ the period is expanded. Again here we take $B \neq 0$ as otherwise the graph would be a constant. If $B = 1$, the period is unchanged. When trying to graph $y = \cot Bx$ you can just alter the vertical asymptotes $y = n\pi$ to be $y = \frac{n\pi}{B}$, then just graph the point on the $x - axis$ and use the asymptotes to keep the shape; also, you can use the points $\left(\frac{\frac{\pi}{4}}{B}, 1\right)$ and $\left(\frac{\frac{3\pi}{4}}{B}, -1\right)$.
- The phase shift C is a horizontal translation C units to the right if $C > 0$, and a horizontal translation C units to the left if $C < 0$. If $C = 0$, the graph remains unchanged. Make sure that you also translate the asymptotes with the graph!
- The vertical translation D moves the graph D units upward if $D > 0$, D units downward if $D < 0$, and if $D = 0$, the graph remains unchanged.

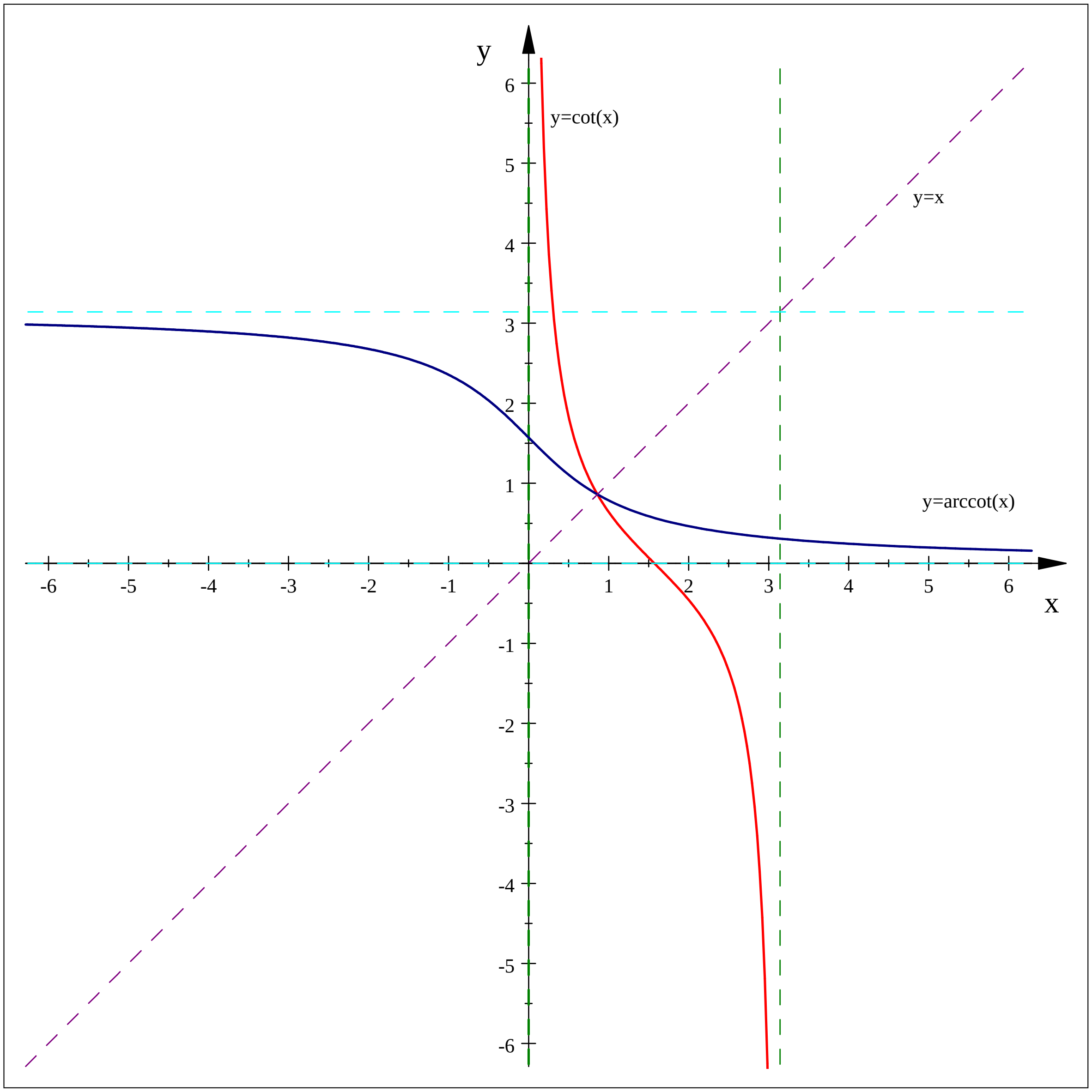
To invert cotangent we have to restrict to a domain where tangent is $one-to-one$. Observe the following graph:



Notice that inside each pair of adjacent asymptotes the graph is $one-to-one$ and decreasing. Consider the middle two asymptotes. So lets ignore everything outside of these asymptotes:



Now the graph is invertible, so to invert it let's draw the diagonal $y = x$ and reflect the above graph (and its asymptotes!) across the diagonal:



Notice that when we restrict cotangent to be one-to-one, the point of intersection with the $x - axis$ stays the same, so we can draw $y = \operatorname{arccot} x$ the same way as $y = \cot x$, using the asymptotes (which are the asymptotes of $y = \cot (x)$ reflected about the line $y = x$). Since an inverse function basically switches the x and y coordinates, the three points above become $\left(-1, \frac{3\pi}{4}\right), \left(0, \frac{\pi}{2}\right), \left(1, \frac{\pi}{4}\right)$ for $y = \operatorname{arccot} x$.

Some properties of $y = \operatorname{arccot} x$:

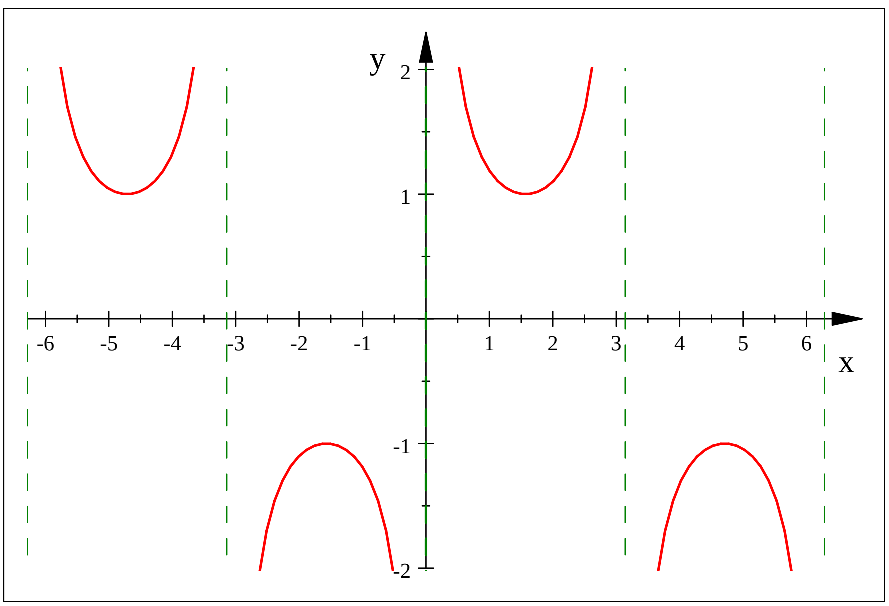
- It is one-to-one.
- It is decreasing.
- It is odd.
- Its domain is $(-\infty, \infty)$.
- Its range is $[0, \pi]$.

5 Cosecant $(y = \csc x)$

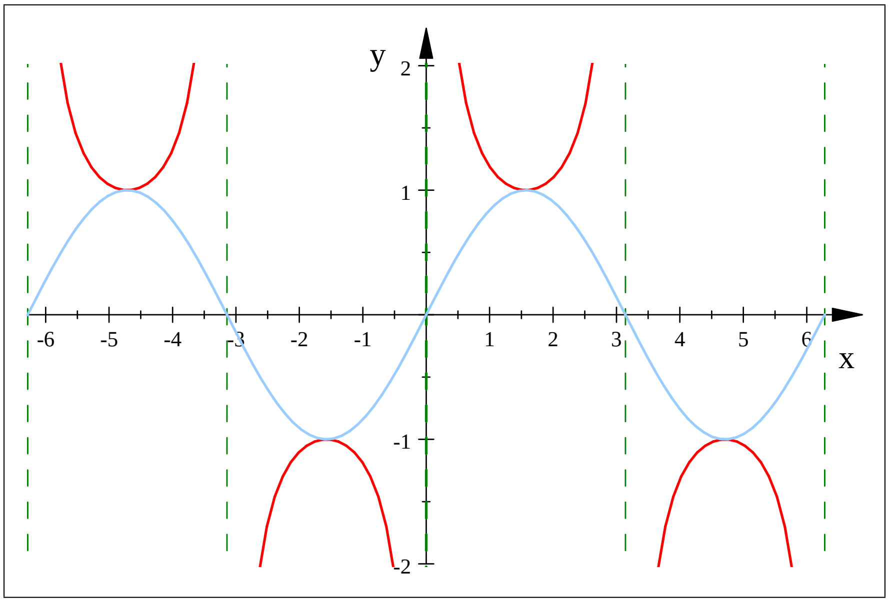
The value of sine at common angles:

$x =$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\csc x =$	$undef$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

The graph of $y = \csc x$ looks like:



Observe when $y = \csc x$ and $y = \sin x$ are plotted on the same graph:



Notice how the graph of cosecant touches the graph of sine where sine has its maximums and minimums. Also notice how the asymptotes of cosecant cross the $x - axis$ at the same points where sine does (this is no coincidence since $\csc x := \frac{1}{\sin x}$, and whenever $\sin x = 0$, $\csc x$ is $undefined$). To graph cosecant, it is easiest to first graph sine, draw the vertical asymptotes where sine crosses the $x - axis$, then draw the "U" shapes in the space between the asymptotes, making sure that the bottom of the U's touch the maximums of sine, and that the tops of the upsidedown U's touch the minimums of sine.

A transformation of $y = \csc x$ is of the following form:

$$y = A\csc\left(B\left(x - C\right)\right) + D$$

where:

- $|A|$ is the amplitude of the sine graph in the middle of the graph cosecant graph,
- the period of the graph is $P = \frac{2\pi}{B}$,
- the phase shift of the graph is C,
- and the vertical translation is D.

When graphing a transformation of cosecant, it is useful to graph $y = A\sin\left(B\left(x - C\right)\right) + D$, then graph $y = A\csc\left(B\left(x - C\right)\right) + D$, using the method described above to graph $y = \csc x$.

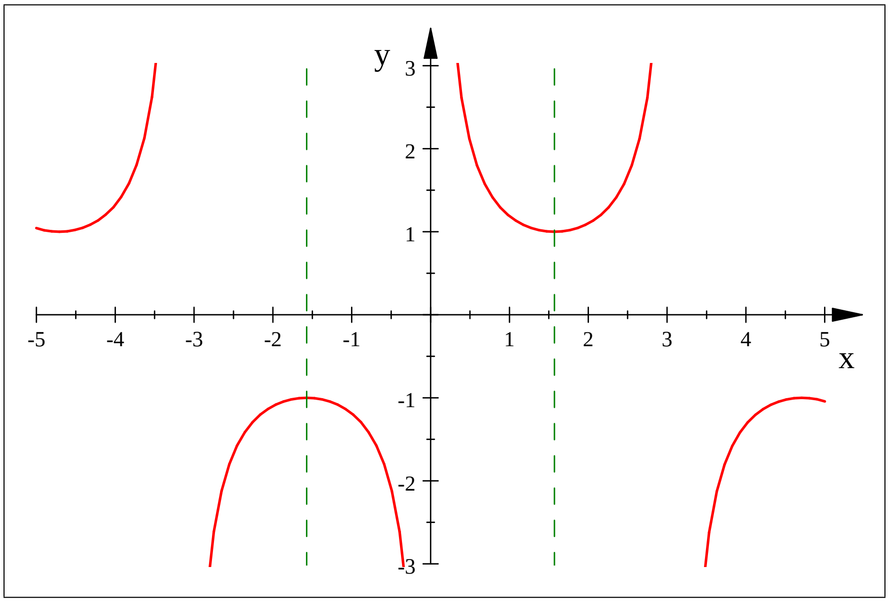
Some facts:

- The domain is $\mathcal{D} = \{x \in \mathbb{R} \mid x \neq n\pi \text{ where } n \text{ is an integer } (..., -2, -1, 0, 1, 2, ...)\}$,
- the range is $(-\infty, -A + D] \cup [A + D, \infty)$.

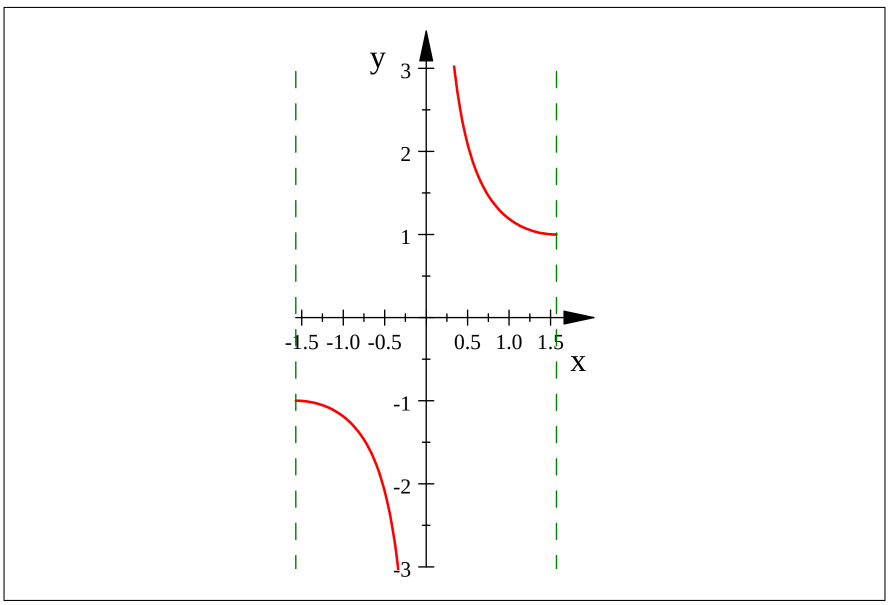
A geometric interpretation of the equation (these descriptions are transformations of $y = \csc x$):

- You can think of the length of the gap between the lower part and the upper part of cosecant as being $2|A|$. If $|A| > 1$, the graph gets stretched taller in the $y - direction$; if $0 < |A| < 1$, then the graph gets shrunk shorter in the $y - direction$; lastly if $|A| = 1$, the graph does not get stretched or shrunk in the vertical direction. (We do not consider the case if $A = 0$ because, if $A = 0$, the graph is just the line $y = 0$.) Lastly, if $A < 0$, then the graph is reflected over the $x - axis$.
- The period is how long the graph takes to complete one cycle. We can assume B to be positive since cosecant is an odd function (i.e. $\csc(-x) = -\csc(x)$). The standard period of cosecant is 2π. If $B > 1$ the period is shrunk, and if $B < 1$ the period is expanded. Again here we take $B \neq 0$ as otherwise the graph would be a constant. If $B = 1$, the period is unchanged.
- The phase shift C is a horizontal translation C units to the right if $C > 0$, and a horizontal translation C units to the left if $C < 0$. If $C = 0$, the graph remains unchanged.
- The vertical translation D moves the graph D units upward if $D > 0$, D units downward if $D < 0$, and if $D = 0$, the graph remains unchanged.

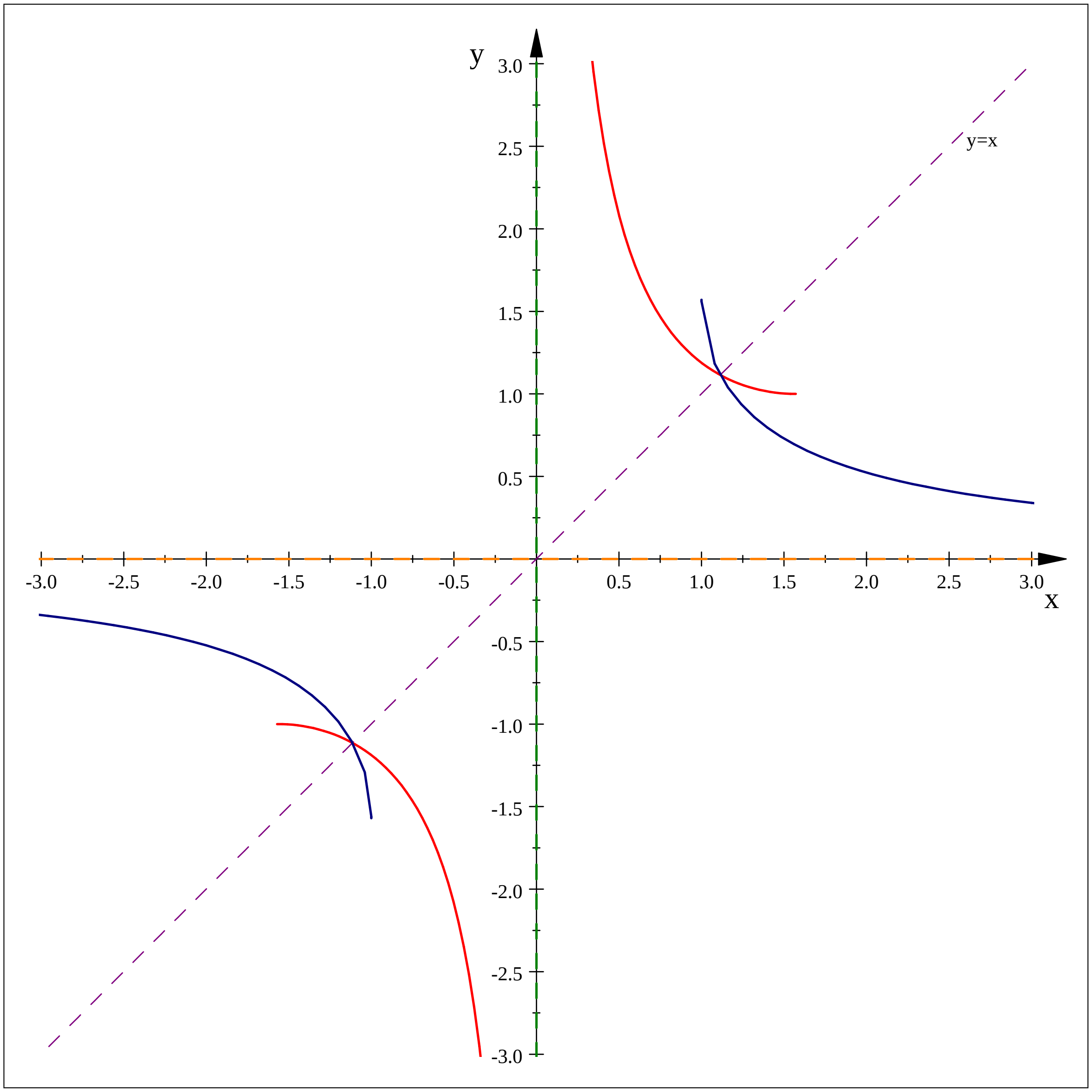
To invert cosecant we have to restrict to a domain where cosecant is $one - to - one$. Observe the following graph:



The green lines above cut the graph at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. Notice that inside these two green lines the graph is $one - to - one$. So lets ignore everything outside of these green lines:



Now the graph is invertible, so to invert it let's draw the diagonal $y = x$ and reflect the above graph and its asymptote across the diagonal ($y = \csc x$ is in red with a green asymptote and $y = \operatorname{arccsc} x$ is in blue with an orange asymptote):



Notice that when we restrict cosecant to be one-to-one, the two points $\left(-\frac{\pi}{2}, -1\right)$ and $\left(\frac{\pi}{2}, 1\right)$ are the two endpoints. Since an inverse function basically switches the x and y coordinates, these two points become $\left(-1, -\frac{\pi}{2}\right)$ and $\left(1, \frac{\pi}{2}\right)$ for $y = \operatorname{arccsc} x$.

Some properties of $y = \operatorname{arccsc} x$:

- It is one-to-one.

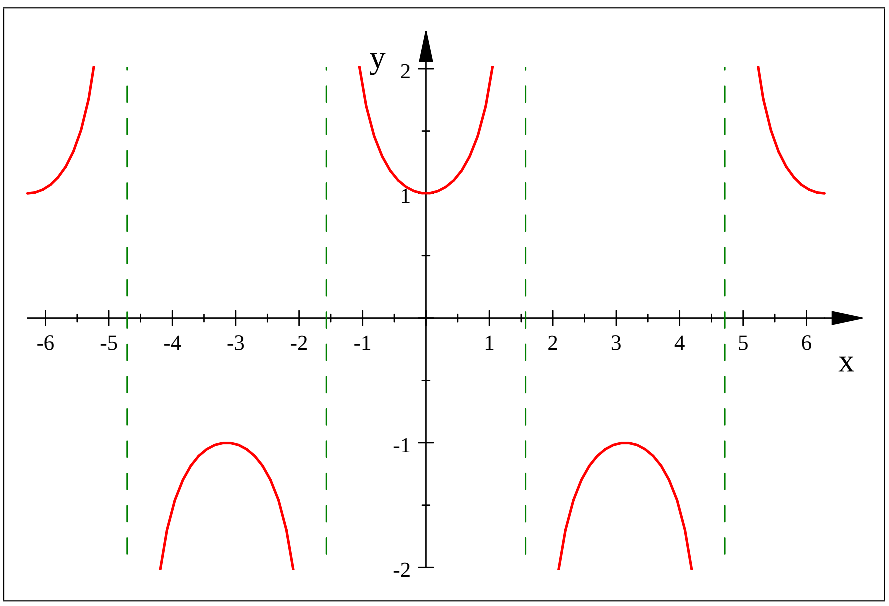
- It is odd.
- Its domain is $(-\infty, -1] \cup [1, \infty)$.
- Its range is $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$.

6 Secant $(y = \sec x)$

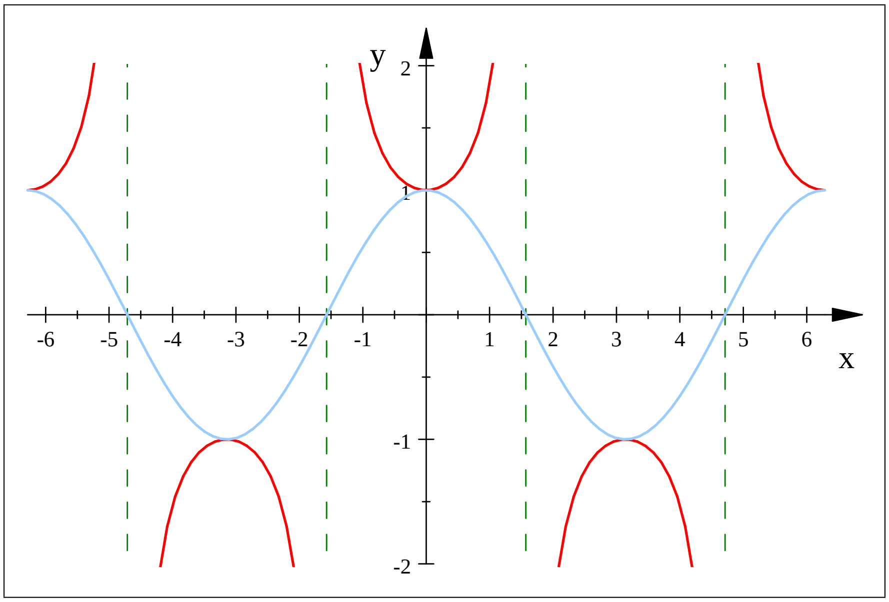
The value of sine at common angles:

$x =$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sec x =$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$undefined$

The graph of $y = \csc x$ looks like:



Observe when $y = \sec x$ and $y = \cos x$ are plotted on the same graph:



Notice how the graph of secant touches the graph of cosine where cosine has its maximums and minimums. Also notice how the asymptotes of secant cross the $x-axis$ at the same points where cosine does (this is no coincidence since $\sec x := \frac{1}{\cos x}$, and whenever $\cos x = 0$, $\sec x$ is $undefined$). To graph secant, it is easiest to first graph cosine, draw the vertical asymptotes where cosine crosses the $x-axis$, then draw the "U" shapes in the space between the asymptotes, making sure that the bottom of the U's touch the maximums of cosine, and that the tops of the upsidedown U's touch the minimums of cosine.

A transformation of $y = \sec x$ is of the following form:

$$y = A \sec\left(B\left(x - C\right)\right) + D$$

where:

- $|A|$ is the amplitude of the cosine graph in the middle of the graph cosecant graph,
- the period of the graph is $P = \frac{2\pi}{B}$,
- the phase shift of the graph is C,
- and the vertical translation is D.

When graphing a transformation of cosecant, it is useful to graph $y = A \cos\left(B\left(x - C\right)\right) + D$, then graph $y = A \sec\left(B\left(x - C\right)\right) + D$, using the method described above to graph $y = \sec x$.

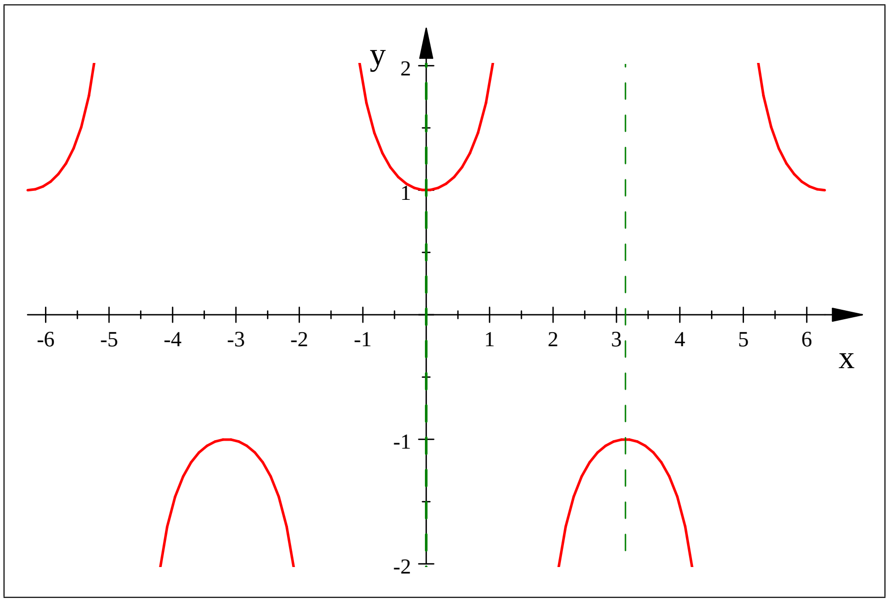
Some facts:

- The domain is $\mathcal{D} = \left\{ x \in \mathbb{R} \mid x \neq \frac{n\pi}{2} \text{ where } n \text{ is an odd integer } (..., -3, -1, 1, 3, ...) \right\}$,
- the range is $(-\infty, -A + D] \cup [A + D, \infty)$.

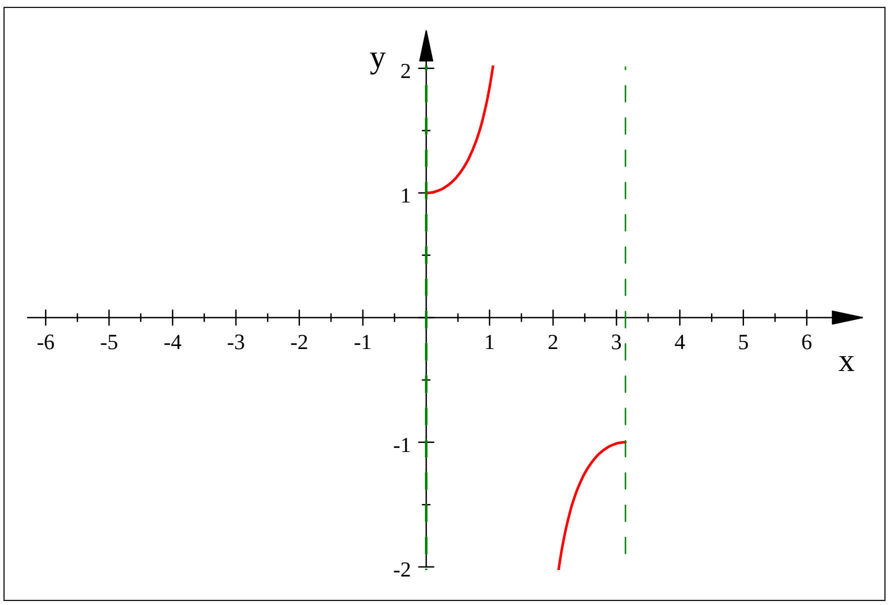
A geometric interpretation of the equation (these descriptions are transformations of $y = \sec x$):

- You can think of the length of the gap between the lower part and the upper part of secant as being $2|A|$. If $|A| > 1$, the graph gets stretched taller in the $y - direction$; if $0 < |A| < 1$, then the graph gets shrunk shorter in the $y - direction$; lastly if $|A| = 1$, the graph does not get stretched or shrunk in the vertical direction. (We do not consider the case if $A = 0$ because, if $A = 0$, the graph is just the line $y = 0$.) Lastly, if $A < 0$, then the graph is reflected over the $x - axis$.
- The period is how long the graph takes to complete one cycle. We can assume B to be positive since cosecant is an even function (i.e. $\sec(-x) = \sec(x)$). The standard period of secant is 2π. If $B > 1$ the period is shrunk, and if $B < 1$ the period is expanded. Again here we take $B \neq 0$ as otherwise the graph would be a constant. If $B = 1$, the period is unchanged.
- The phase shift C is a horizontal translation C units to the right if $C > 0$, and a horizontal translation C units to the left if $C < 0$. If $C = 0$, the graph remains unchanged.
- The vertical translation D moves the graph D units upward if $D > 0$, D units downward if $D < 0$, and if $D = 0$, the graph remains unchanged.

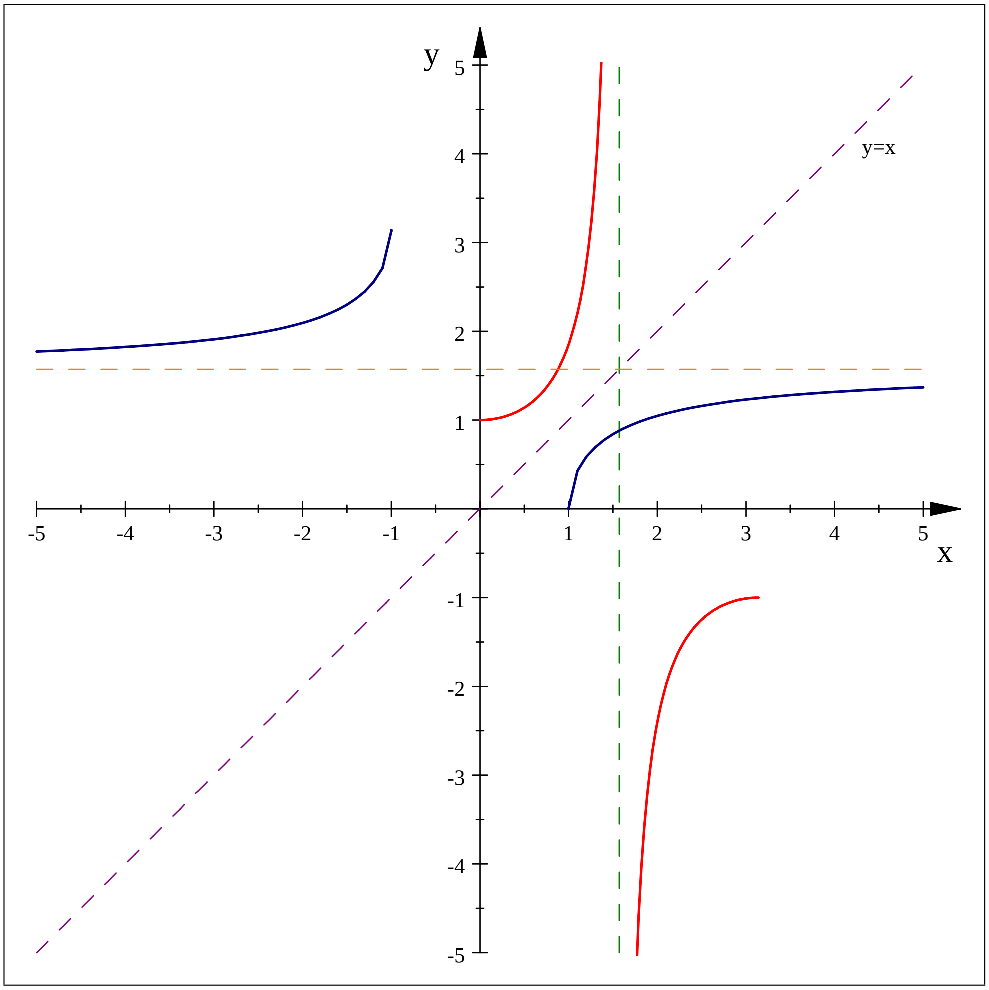
To invert secant we have to restrict to a domain where secant is $one{-}to{-}one$. Observe the following graph:



The green lines above cut the graph at $x = 0$ and $x = \pi$. Notice that inside these two green lines the graph is $one - to - one$. So lets ignore everything outside of these green lines:



Now the graph is invertible, so to invert it let's draw the diagonal $y = x$ and reflect the above graph and its asymptote across the diagonal ($y = \sec x$ is in red with a green asymptote and $y = \operatorname{arcsec} x$ is in blue with an orange asymptote):



Notice that when we restrict cosecant to be one-to-one, the two points $(0, 1)$ and $(\pi, -1)$ are the two endpoints. Since an inverse function basically switches the x and y coordinates, these two points become $(1, 0)$ and $(-1, \pi)$ for $y = \operatorname{arcsec} x$.

Some properties of $y = \operatorname{arcsec} x$:

- It is one-to-one.
- Its domain is $(-\infty, -1] \cup [1, \infty)$.
- Its range is $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$.